

10

PROBABILITIES AND THE FINE-TUNING ARGUMENT

A skeptical view

Timothy McGrew, Lydia McGrew, and Eric Vestrup

Contemporary cosmological design arguments consist broadly of two main types: the Life Support Argument (LSA), which urges that the relatively local features of the cosmos (our Galaxy, the Sun, the Earth, the Moon) are unusually hospitable to life, and the Fine-Tuning Argument (FTA), which takes its cue from the fact that a multitude of physical constants must apparently take precise values or stand in exacting ratios to each other in order to make our universe as a whole hospitable to organic life. The apparent “fine-tuning” of the constants suggests to the advocates of the FTA that the Universe was itself designed with carbon-based life in mind.

Much of the critical discussion of the FTA in the current literature centers on two factors. First, some critics question the material adequacy of the argument: whether the relevant physical constants are indeed related in such exacting ratios or can be relaxed while remaining, in some sense, life-friendly (Shapiro and Feinberg 1982; Barrow and Tipler 1986). Second, some suggest that the argument derives specious plausibility from a selection effect. Life-friendly universes, so this criticism runs, may be rare, but the existence of conscious life in life-friendly universes is not such a puzzle. Like the lucky winner of a lottery who finds himself bemused and wonders if there has been some mistake, the denizens of a life-friendly universe who marvel at their own good fortune are forgetting that anyone else would find the situation equally astonishing (Carter 1974).

These are interesting arguments, but they leave the formal aspects of the FTA unquestioned. By contrast, we have serious doubts about the use of probabilities in the FTA—in particular, the calculation of the odds against a life-friendly universe “on chance.” This phrase, as a moment’s reflection reveals, cannot have an obvious, everyday meaning: the image of a barrel full of universes from which one is selected at random does not inspire confidence. In the literature the talk of probabilities is generally supported with analogies and stories designed to pump the intuition in favor of the FTA. Many of these stories describe plausible evidence for inferring design or agent intervention, but we contend that they are crucially disanalogous to the problem of the origin of the Universe “on chance.”

The FTA, as it stands in the literature, has serious formal flaws that may well be insuperable.

The structure of the argument

The literature on the FTA overflows with examples of “fine-tuned” constants (Leslie 1989, 1998; Ross 1998), and it would be superfluous to survey them here. Typically, however, they take the form of placing upper and lower bounds on the constants in question. Advocates of the FTA assume, plausibly enough, that the list of constants, and hence of their ratios, is finite, though there is some argument as to whether the various parameters are independent of each other. In each case, the field of possible values for the parameters appears to be an interval of real numbers unbounded at least in the upward direction. There is no *logical* restriction on the strength of the strong nuclear force, the speed of light, or the other parameters in the upward direction. We can represent their possible values as the values of a real variable in the half-open interval $[0, \infty)$, and the set of logically possible joint values for K independent constants can be represented as a K -dimensional real-valued space \mathbb{R}_+^K . If we consider each mathematically distinct set of possible values for the parameters to denote a distinct possible type of universe, then we can think of this space as representing all of the possible types of universes that have no types of constraints other than those found in ours.

Among the universes thus represented, which ones will be friendly to life? There is some vagueness surrounding this question, since it is not clear just what life-friendliness amounts to (Manson 2000). At least one universe qualifies: the point that represents the various constants actually exhibited in our universe is clearly life-friendly. But in some of the alternative universes considered—say, those in which the strength of the gravitational force is slightly greater—the upshot is not to rule out life absolutely but rather to lessen the odds of its arising and surviving.

The point can, however, be waived: let us grant that there is a plausible convention we may adopt as to the line of demarcation between life-friendly and life-unfriendly universes, and that this convention will give us a range, perhaps even a narrow one, within which each variable will have to fall in order for the universe to be life-friendly. It seems quite plausible (and seems to be tacitly granted in discussions of the FTA) that, for each constant, the life-friendly region will have an interior of positive Lebesgue measure—that the “inside” of the region will have positive “volume.” These regions need not satisfy any other conditions for our discussion. However, for graphical simplicity, we can without loss of generality treat the points lying within these intervals in all K dimensions as what mathematicians affectionately term a “ball”—a set that includes the

points representing our universe and those universes sufficiently similar to ours to meet the criterion we have adopted.¹

Might the life-friendly region end up being just the singular point corresponding to our own universe? This seems extremely unlikely. Since the variables in question range over the reals, we may reasonably assume that there are infinitely many possible universes that are arbitrarily similar to ours though mathematically distinct—universes in which the constants differ from those in ours by amounts so small that the physical implications remain negligible even in the large-scale effects. In such universes, admittedly, some highly elegant symmetry principles and conservation laws might be violated (though the violations could be too small to be physically detected), but these principles appear to be contingent rather than logically necessary. It is certainly incumbent on anyone who would contest this possibility to explain why *only* this universe, and not one arbitrarily similar to it, could sustain life.

With this model in view, we can state the FTA in a more rigorous form. Assume for the sake of the argument that no particular range of values for the parameters is more likely than any other—an application of the Principle of Indifference designed to reflect our lack of information regarding universes. The measure, in R_+^K , of the multi-dimensional “ball” is vanishingly small: like a grain of sand lost in an endless Euclidean universe, it takes up no finite proportion of the possible space.² If the measure of the ball represents the probability in question, then it would seem that the odds against a life-friendly universe on chance are, to put it mildly, overwhelming. Advocates of the FTA maintain that the odds of a life-friendly universe under the creative auspices of an intelligent designer are a good deal better. But in that case, so runs the argument, the evidence for the fine-tuning of the Universe must count as evidence favoring design over chance.

In his delightful book *Universes*, John Leslie tries to illustrate the form of the argument by telling stories in which our suspicions are aroused because chance provides a poor explanation for the coincidences at hand. In the Fishing Story, to which he returns many times throughout the book, you catch a fish that is exactly 23.2576 inches long from a murky lake. In itself this is not very surprising: no doubt every fish needs to have some length. But then you note that your fishing apparatus was capable of catching only fish of this length, plus or minus one part in a million. You might hypothesize many explanations for this astonishing coincidence—that the lake is stuffed with so many different sizes of fish that you were bound to find one that fit sooner or later; that a benevolent deity desired to provide you with a fish supper; and so forth. But surely, says Leslie, one explanation that should be ruled out at once is the idea that the lake contained only one fish, which *just happened* to be the ideal size for you to catch. As with fish, so with universes: there may be many universes, of which ours is a singularly life-friendly specimen; there may be a benevolent creator who desired

to create a universe that could sustain life. But one explanation Leslie thinks we should certainly reject is the notion that our universe is the only one and just happened, by chance, to be so constituted as to sustain life (Leslie 1989: 9ff.).

The normalizability problem

The Fishing Story solicits our assent by describing a situation where our intuitive judgment of the relevant probabilities leans heavily against chance. But before we can port those intuitions over to the FTA, we need to examine the mathematical model in more detail. When we do so, it turns out that the analogy between the FTA and the Fishing Story breaks down—and it breaks down precisely where probabilities are invoked.

The critical point is that the Euclidean measure function described above is not normalizable. If we assume every value of every variable to be as likely as every other—more precisely, if we assume that, for each variable, every small interval of radius ϵ on \mathbb{R} has the same measure as every other—there is no way to “add up” the regions of \mathbb{R}_+^K so as to make them sum to one. If they have any sum, it is infinite.

This is more than a bit of mathematical esoterica. Probabilities make sense only if the sum of the logically possible disjoint alternatives adds up to one—if there is, to put the point more colloquially, some sense that attaches to the idea that the various possibilities can be put together to make up 100 percent of the probability space. But if we carve an infinite space up into equal finite-sized regions, we have infinitely many of them; and if we try to assign them each some fixed positive probability, however small, the sum of these is infinite.

Unfortunately, there is not a great deal of discussion on this point in the philosophical literature since the normalization problem has not been raised with sufficient clarity. Leslie does consider some worries about ratioing infinite quantities, but he deflects them with an analogy. The bull’s-eye of a target, he points out, contains infinitely many mathematical points (as does, he might have added, the rest of the target). Since this fact makes it no easier to hit the bull’s-eye, Leslie dismisses concern about his use of probabilities as mathematical pettifogging (Leslie 1989:11).

But in view of the foregoing discussion we can see that this analogy is flawed. The target itself has finite area; we can therefore integrate over the area and compare regions without running into normalizability problems. Leslie clearly does not recognize the way in which the bull’s-eye and the realvalued parameter space are disanalogous, for he goes on in the next paragraph to speak of life-permitting possibilities constituting “only a thousandth of the range of possibilities under consideration”—an expression that only makes sense because normalizability is not a problem with the target.

What moves us to adopt such a picture in the first place? The culprit is the Principle of Indifference. Working from bare logical possibilities, it seems unreasonable to suggest that any one range of values for the constants is more probable *a priori* than any other similar range—we have no right to assume that one sort of universe is more probable *a priori* than any other sort. But this very feature rules out this measure function as a basis for assigning probabilities.

The problem is not simply that there are infinitely many points both within the ball and outside of it: there are mathematical techniques for coming to grips with that problem. The difficulty lies in the fact that there is no way to establish ratios of regions in a non-normalizable space. As a result, there is no meaningful way in such a space to represent the claim that one sort of universe is more probable than another. Put in non-mathematical language, treating all types of universes evenhandedly does not provide a *probabilistic* representation of our ignorance regarding the ways that possible universes vary among themselves—whatever that means.

Rescues and replies

One way to try to get around the problem is simply to give up a probabilistic interpretation of the narrow intervals and rely instead on an intuitively plausible argument from extrapolation. A small area in a larger finite region, like the bull's-eye at the center of a target, is relatively less likely to be hit (at random) as its size diminishes in relation to the rest of the target. If we think of an *infinitely* large target with a finite bull's-eye, we seem to have the limiting case of low probabilities. Even if we cannot represent this as a ratio of areas in a strict probabilistic sense, are we not entitled to take the “ratio” of a finite to an infinite measure as a basis for the FTA?

Unfortunately, this rescue, if permitted, would achieve far too much. For using such reasoning we can also underwrite what we shall call the “Coarse-Tuning Argument” (CTA). Suppose that the open set of life-friendly universes contained a ball in which the various parameters, rather than being constrained to within tiny intervals around those that characterize our own universe, could take any values within a few billion orders of magnitude of our values. It is hard to imagine anyone's being surprised at the existence of a life-friendly universe under such circumstances. Yet the “ball” in this case is isomorphic to the ball in the FTA: both of them have measure zero in \mathbb{R}_+^K . In consequence, any inference we can draw from fine-tuning is not only paralleled by a CTA, it also has precisely the same probabilistic force. So if we are determined to invoke the Principle of Indifference regarding possible universes, we are confronted with an unhappy conditional: if the FTA is a good argument, so is the CTA. And conversely, if the CTA is not a good argument, neither is the FTA.

A natural response to this is to blame the Principle of Indifference. If we do not insist on treating all equal intervals for all parameters as equally probable, then we can perfectly well speak of the probability that a particular parameter falls within a given interval by invoking density functions that integrate (or can be scaled so as to integrate) to unity. Loosely speaking, this is similar to using a converging series in which an infinite number of terms (say, $1/2, 1/4, 1/8, \dots, 1/2^n, 1/2^{n+1}, \dots$) may have a finite sum (in this case, 1), because there is no finite positive lower bound on the size of the terms. By the standard mathematical device of taking the probability in question to be proportional to the area beneath that region of the density function, we can bring the FTA back within the pale of respectable probability theory.

And so we can. But which density function shall we now choose; which intervals shall we favor? There are myriads, continua, of such functions. What is worse, there are infinitely many density functions that pack as great a proportion of the probability mass as you like into the ball; and from the standpoint of any of these functions the existence of a life-friendly universe on chance is practically *inevitable*. True, there are also infinitely many density functions that render the measure of the ball arbitrarily small. But now we have traded our initial problem for another: how to choose, without arbitrariness, which density function or family of density functions is the “right” one. And every such function will be biased in favor of some regions over others—a bias that is difficult to justify in a representation of the bare possibilities.

How does this analysis square with the intuitions prompted by the Fishing Story? Fish, as many a lazy summer’s day out on the lake has taught us, vary widely in size: their mature lengths are approximately normally distributed, and relatively few are within one part in a million of 23.2576 inches—or one part in a million of any other specific length. From such homely data we construct reasonable expectations regarding the lengths of fish, and it is those expectations that give the example its plausibility. The data mitigate against any expectations that render a length of 23.2576 inches, plus or minus a few microns, overwhelmingly likely for the next fish we shall catch. But such data are precisely what we lack regarding universes.

This illustrates a second way in which Leslie’s bull’s-eye analogy is misleading. Thanks to our ample experience of projectile motion, we have a non-arbitrary means of adjudicating disputes regarding non-equivalent density functions over the target. Experience indicates that a flat prior is reasonable for randomly flung darts—any area of a given size is as likely to be hit as any other if the thrower is sufficiently unskilled.

An important attempt to salvage the FTA can be found in Leslie’s Fly on the Wall Story (Leslie 1989:17–18; cf. 158ff.). Suppose that a fly occupies an otherwise vacant stretch of a wall. Perhaps far away there are regions densely covered with flies, but in the local area around this fly the wall is clear. A shot

rings out and the lonely fly is struck. Surely, argues Leslie, this requires explanation regardless of the presence or absence of many flies far away. For *in the area of interest* there was a very small chance of hitting a fly at random. Carried over to the FTA, the Fly on the Wall Story is a plea for working with a normalizable space.

The Fly on the Wall Story can be pressed into service for two distinct purposes. On the one hand, the wall may represent a space of possible *types of parameters*.³ Obviously, if we stray far from the types of parameters we find in our own universe we may not be in any position to speak of the life-friendliness of universes. (Would a small repulsive force between photons be life-unfriendly? What if it were combined with tweaks of half a dozen other constants and a few utterly new forces?) From this standpoint, our willingness to restrict the discussion to R_+^K rather than introducing a possibly infinite number of new parameters represents an attempt to take the Fly on the Wall Story seriously.

On the other hand, the wall in Leslie's story may represent a range of values for the parameters we actually have. In this case, however, there is a serious difficulty in determining how wide a range we ought to survey, how much of the "wall" we ought to take into account. On what basis should we restrict our focus to the area that is amenable to current theoretical discussion? There is, of course, a good pragmatic argument for discussing only those possible universes whereof we are, in some sense, qualified to speak. But there is a serious gap between this sensible pragmatic advice and the epistemic force that the FTA is supposed to have. What we need is an *epistemic* rationale for working with the local region rather than the whole of R_+^K . Without such a restriction, we are back to the normalizability problem. And no one has yet succeeded in articulating a convincing reason for limiting the field.

Some proponents of the FTA will object that all of this is needlessly fussy, that we need no airtight philosophical criterion in order to see that any function that does not make the anthropic coincidences surprising is an unreasonable function.⁴ It is certainly true that a number of people not otherwise known for their theological interests have been greatly impressed with the apparent narrowness of the intervals for life-friendly universes. But in our opinion it is profoundly unsatisfying to stipulate that we can just "tell" which functions are reasonable and which are not. Reasonable people have conflicting intuitions here. The point of the argument was supposed to be that objective results in modern cosmology virtually compel disbelief in a chance origin of the Universe. If, at a critical point, the argument turns on a subjectively variable sense of which assessments of probabilities are reasonable, a sense that cannot be adjudicated in terms of any more fundamental criteria, then the FTA is effectively forceless. To retreat to the point where the argument rests on unargued intuitions is to deprive it of anything more than devotional significance.

A wholly different approach involves appealing to inflationary cosmology to give us an empirical probability distribution over universes. Waiving questions about the empirical affidavits of inflationary schemes, however, this will give advocates of the FTA no comfort, for two reasons. First, the probabilities generated all depend on some very strong extrapolations of indifference assumptions, extrapolations exorbitant enough to warrant some skepticism. Second and more importantly, in inflationary scenarios the “universes” in question are all physically real; ours is just one universe among an enormous number of others that are separating from each other faster than they are expanding. But this undermines the need to appeal to design in the first place: the appearance of design is just a feature of our “universe,” which is a small but statistically predictable outlier among the vast horde of coexisting “universes.”

It is natural to wonder whether a critique of this kind can be ported over to the LSA, thereby demolishing in one fell swoop both types of cosmological design argument.⁵ In our opinion these particular problems do not necessarily affect the LSA. If the prior probability of a hospitable planet on chance is taken to be, for example, the odds that a randomly selected star will have a planet of the appropriate mass, axial tilt, and so forth—in short, conditions suitable for the flourishing of life—then there is no reason in principle that these odds cannot be calculated from astronomical data. We do not have to arrive at such odds by direct appeal to the Principle of Indifference.

There are, of course, additional questions that need to be addressed if the LSA is to be convincing, most urgently questions regarding the likelihood of conditions hospitable to carbon-based life in particular given the existence of a designer. But the difference between the two arguments illustrates nicely the distinction between calculating the odds of encountering by chance a certain type of object *within* a universe and calculating the odds on chance of a universe itself.

If universes were as experimentally tractable as amino acids or as profusely displayed to our gaze as binary star systems, then we might in principle be able to collect sufficient statistical information to make informed claims about the relative frequency of life-friendly parameters. But they are not. In consequence, we are in no position to speak of what we might have expected instead of the universe we have. Arguments for design have to be framed within our universe and in terms of its laws if they are to have probative force.⁶

Notes

- 1 The mathematics will not be affected if the points constituting the boundary of the ball are taken to be external to the region; the only crucial assumption here is that these regions have an interior with positive measure.

It is conceivable that the life-friendly zone might take a topologically more interesting form if the loss of life-friendliness contingent on increasing the value of one parameter could be restored by increasing or decreasing others. We might then have a life-friendly “foam” in R_+^K , a continuous region of universes favorable to life but sensitive to the alteration of individual parameters. This possibility tends, however, to lessen the force of the FTA by increasing the range of possible life-friendly universes. In what follows we will ignore it.

- 2 The argument could, of course, have some force even if the probability of a life-friendly universe on the chance hypothesis were merely somewhat low rather than vanishingly small, provided that it is lower than the probability on the design hypothesis.
- 3 Leslie seems to endorse this interpretation (1989:17).
- 4 We owe this objection to Rob Koons in extended correspondence.
- 5 John Leslie points out in private conversation that another version of what might be called an FTA, in which the ratio of two constants encodes a detailed message, may also escape the normalizability problem. This argument raises fresh issues and lies beyond our scope here, but it is worth pointing out that no one has seriously advanced the claim that the ratios of the constants actually encode such a message.
- 6 Thanks to John Leslie for constructive criticism that improved the paper, Robin Collins for discussions of inflationary cosmology, Rob Koons for much vigorous correspondence over the FTA, and the participants at the two places where earlier versions have been presented: the “Design and its Critics” conference at Concordia University in June 2000 and the Notre Dame summer symposium on Cosmic Fine-tuning in July 2000, particularly Neil Manson, Mike Thrush, John Mullen, Bradley Beach, and Peter and Alisa Bokulich.

References

- Barrow, J. and Tipler, F. (1986) *The Anthropic Cosmological Principle*, Oxford: Clarendon Press.
- Carter, B. (1974) “Large number coincidences and the anthropic principle in cosmology,” in Leslie, J. (ed.) (1998), pp. 131–9
- Leslie, J. (ed.) (1998) *Modern Cosmology and Philosophy*, New York: Prometheus Books.
- (1989) *Universes*, New York: Routledge.
- Manson, N. (2000) “There is no adequate definition of ‘fine-tuned for life,’” *Inquiry* 43: 341–51.
- Ross, H. (1998) “Big Bang model refined by fire,” in W.Dembski (ed.) (1998) *Mere Creation*, Downer’s Grove, Illinois: InterVarsity Press, pp. 363–84.
- Shapiro, R. and Feinberg, G. (1982) “Possible forms of life in environments very different from earth,” in Leslie, J. (ed.) (1998), pp. 254–61 .